Appendix: Proofs

A 12-character expanded name is coded into a number base 27, call it N. N is of the form

$$N = a_{12} \times 27^{12} + \cdots + a_i \times 27^i + \cdots + a_1 \times 27 + a_0$$

where the a_i coefficients are all in the range $[0, 1, \ldots, 26]$. We must show that common errors always change the remainder when N is divided by 29. We consider single-character errors and transpositions.

single character error

Suppose a single character of N is in error, i.e., say a_i has been mistakenly entered as b_i, so that

$$N' = a_{12} \times 27^{12} + \cdots + b_i \times 27^i + \cdots + a_1 \times 27 + a_0$$

Then

$$N - N' = (a_i - b_i) \times 27^i$$

We note that 29 cannot divide $a_i - b_i$, which is at most 26 (range from -26 to $+26$); nor, since it is a prime, can 29 divide 27^i, which has 3 as its only prime factor. Therefore it does not divide $N - N'$, and so $N' \neq N \bmod 29$.

transpositions

Suppose there is a transposition of two letters, corresponding to a transposition of two coefficients, a_i and a_j, where $i > j$. Now $N - N'$ will be of the form

$$\begin{aligned} N - N' &= (a_i - a_j) \times 27^i + (a_j - a_i) \times 27^j \\ &= (a_i - a_j) \times 27^i - (a_i - a_j) \times 27^j \\ &= (a_i - a_j) \times 27^j \times (27^{i-j} - 1) \end{aligned}$$

As for "single character error", above, 29 cannot divide either of the first two factors. We must examine whether 29 can divide the third, i.e., a number of the form $27^k - 1$, where k is an integer in the range 1–11. This is a matter of simply testing all 11 values of k, one at a time, and none of these is divisible by 29. (Actually this is the case for any k in the range 1–27.) So once again, the error results in an N' that has a different remainder from N, mod 29. (And, indeed, the method would work for strings up to 28 characters).

other random errors

It is possible that multiple errors would compensate for each other, giving a false validity check. The frequency of false checks, in cases of completely random garble, would be about $1/29 = 3.4\%$. Thus this method detects over 96% of errors, including all of the most common ones.